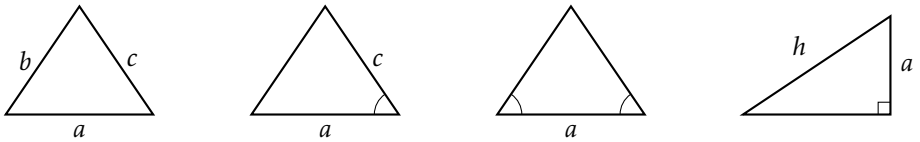
Congruent Triangles

Things you should already know

- Two shapes are **congruent** if they are exactly the same shape and size
- Congruent shapes can be reflections or rotations of each other
- Angles in a triangle sum to 180°

Fact — Two triangles are **congruent** if any one of the following conditions holds:

- **SSS** — all three pairs of sides are equal
- **SAS** — two pairs of sides and the **included** angle are equal
- **ASA** (or AAS) — two pairs of angles and a corresponding side are equal
- **RHS** — both have a right angle, equal hypotenuses, and one other equal side



SSS **SAS** **ASA** **RHS**

Example

Why is **SSA** (two sides and a non-included angle) **not** sufficient for congruence?

Example

Why is **AAA** not sufficient for congruence?

Example

State the congruence condition for each pair:

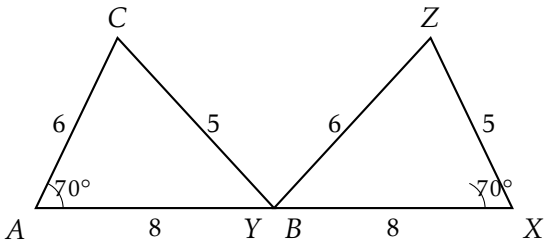
(a) $\triangle ABC$ with $AB = 5$, $BC = 7$, $AC = 8$ and $\triangle DEF$ with $DE = 5$, $EF = 7$, $DF = 8$.

(b) $\triangle PQR$ with $PQ = 6$, $\angle Q = 90°$, $QR = 8$ and $\triangle XYZ$ with $XY = 6$, $\angle Y = 90°$, $YZ = 8$.

(c) $\triangle ABC$ with $\angle A = 40°$, $AB = 9$, $\angle B = 70°$ and $\triangle DEF$ with $\angle D = 40°$, $DE = 9$, $\angle E = 70°$.

Example

Write corresponding vertices in the correct order.



Textbook Exercises: SPSC: 4.3 — Similar Triangles (section on congruence).
Dr Frost: Congruent Triangles slides.
Corbett Maths: Congruent Triangles practice questions (Video 72).
Maths Genie: Congruent Triangles worksheet.
Maths4Everyone: Congruent Triangles booklet.

Similar Triangles

Example

A triangle has angles 40°, 60°, 80° and sides 5 cm, 6 cm, 7 cm. Another triangle has angles 40°, 60°, 80° and sides 10 cm, 12 cm, 14 cm. Are they congruent? What is special about them?

Fact — Two triangles are **similar** if any one of the following conditions holds:

- **AAA** — two (and hence all three) pairs of angles are equal
- All three pairs of sides are **in the same ratio**
- Two pairs of sides are in the same ratio and the included angles are equal

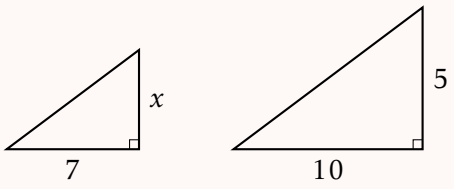
If two triangles are similar, then all corresponding sides are in the same ratio (the **scale factor**).

Finding Unknown Lengths

There are two methods:
Method 1 — Between triangles: compare corresponding sides across the two triangles.
Method 2 — Within triangles: compare sides within the same triangle.

Example

The triangles below are similar. Find x.



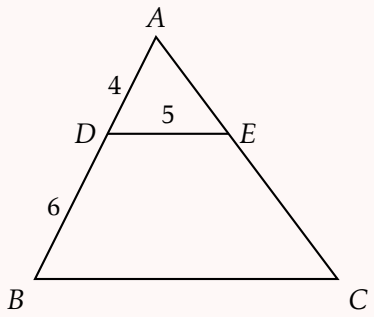
Spotting Similar Triangles

Fact — Look for similar triangles when you see:

- Parallel lines cutting through two sides of a triangle (by alternate/corresponding angles)
- Triangles sharing an angle, with other angles equal
- "Butterfly" configurations (two triangles sharing a vertex)

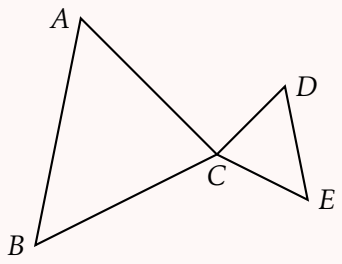
Example

In the diagram, DE is parallel to BC. $AD = 4$ cm, $DB = 6$ cm, $DE = 5$ cm. Find BC.



Example

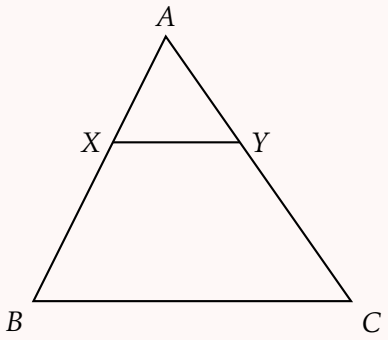
In the diagram, $\angle ACB = \angle DCE$. $AC = 12$, $BC = 8$, $CD = 6$, $CE = 4$. Show that $\triangle ABC$ is similar to $\triangle DEC$.



Internal and External Ratios

Example

In the diagram, $XY \parallel BC$. $AX : XB = 2 : 3$. If $BC = 15$ cm, find XY.



Textbook Exercises: SPSC: 4.3 — Similar Triangles (sections 2–3).
Dr Frost: Similar Shapes slides — similar triangles section.
Corbett Maths: Similar Shapes practice questions (Video 312, 313).
Maths Genie: Similar Shapes worksheet (area and volume scale factors).
Maths4Everyone: Similar Triangles booklet.